Covariance Estimates for Mutual Information Registration

Registration is the task of aligning a pair or group of images of the same region, acquired with some global misalignment, such that voxels in the aligned images correspond to the same point in the scene. It forms a basic component of most medical image analysis algorithms that involve comparisons across multiple image volumes, for example multi-modality image segmentation, image-based monitoring of disease progression, and construction of shape models across whole populations of subjects. Any registration algorithm requires three core components: a similarity metric, a transformation model, and an optimisation algorithm. Registration is then performed by applying the transformation to one image (the source or floating image), and optimising its similarity to the other image (the target or reference image).

In recent years the most common similarity metric used in medical image registration, particularly where multimodality images are involved, is the mutual information measure. This measure is widely believed to be derived from information theory, due to its links to Shannon Entropy. However, in recent work we have shown that mutual information is in fact a biased maximum likelihood technique (see References). Elucidating the link between mutual information and standard statistical procedures has several beneficial consequences. Additional insight is gained into the theoretical origins and statistical properties of the algorithm. This may allow the identification of equivalent but unbiased measures based on quantitative statistics. However, the most important consequence is that it allows the application of the standard error estimation technique for maximum liklihood, namely calculation of the minimum variance bound, producing estimates of the errors on the optimised transformation model parameters. In fact, no quantitative use of the registration result can be made without such error estimates.

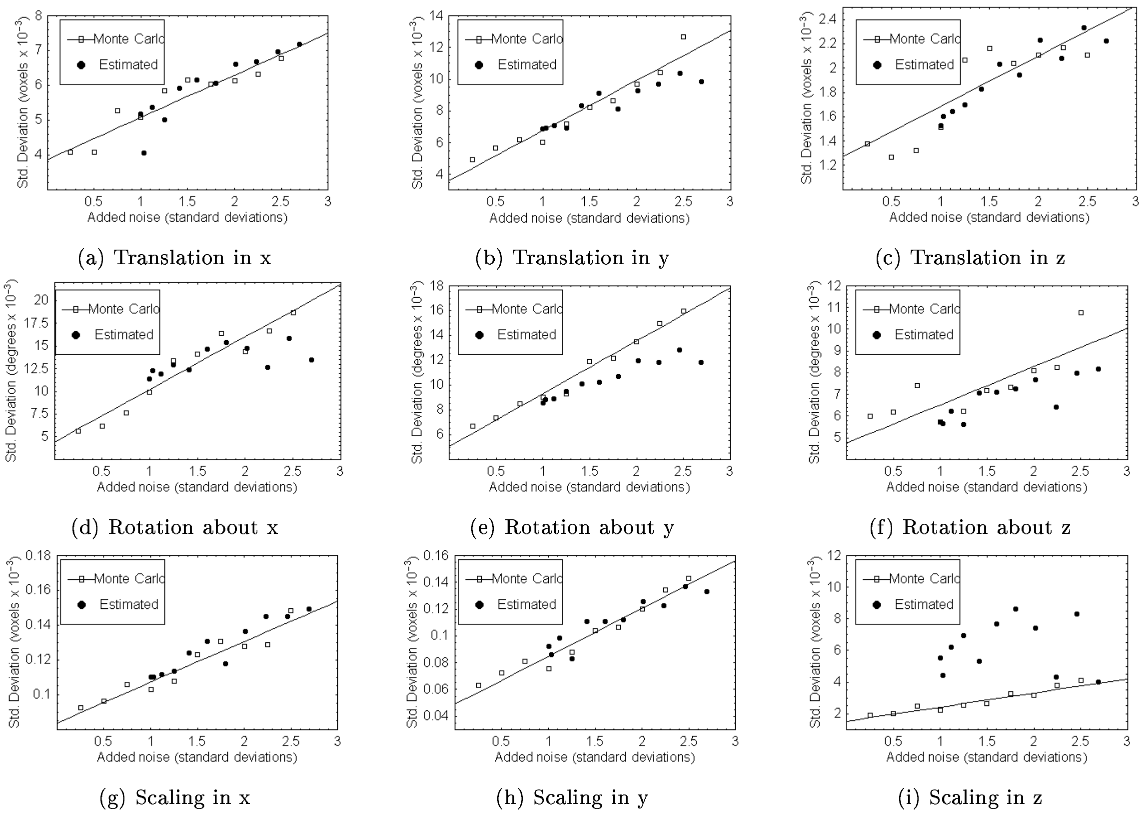
In order to confirm this interpretation, a mutual-information base rigid registration algorithm was implemented in TINA. The transformation model consisted of nine parameters: three each for translation, rotation and scaling. A Monte-Carlo study was then performed in which a pair of MR image volumes (one IRTSE and one T2-weighted) of a normal subject were repeatedly registered. Random Gaussian noise was added to both images prior to each registration. One thousand registrations were performed at each of ten levels of added noise, and the optimised transformation model parameters were recorded. The covariances at each noise level were calculated in the usual way, providing measurements of the errors achieved in practice. Finally, the minimum variance bound was applied to the maximum likelihood interpretation of the mutual information measure in order to produce the theoretical errors. Figure 1 shows the results. A close correspondence between the two quantities can be seen for all parameters except for the scaling in the z direction, where low information content prevented the calculation of a stable theoretical error estimate. A more quantitative comparison is provided by the Bland-Altman plot shown in Fig. 2. This shows the difference between the Monte-Carlo and theoretical measures in units of standard deviation (i.e. the error on the error measurement), plotted against the average of the two measures. Once the data had been presented in this form, paired t-tests were applied across the grouped translation, rotation and scaling results to calculate the probability of equivalence between the theoretical and measured results. The t-test probabilities are given in Table 1. None of the probabilities fall below 5%, showing that there is no statistically significant difference between the measured and theoretical errors, and demonstrating in turn that the interpretation of the mutual information measure as a biased maximum likelihood is valid.

References

[1] Bromiley, P.A., Thacker, N.A., Scott, M.L.J., and Crum, W.R. Empirical Validation of Covariance Estimates for Mutual Information Coregistration. Submitted to IEEE Trans. Med. Imag.

[2] Bromiley, P.A., Pokric, M., and Thacker, N.A., Empirical Evaluation of Covariance Estimates for Mutual Information Coregistration. Proc. MICCAI 2004, Rennes/Saint-Malo, France, 26-30 September 2004, pp. 607-614.

[3] Bromiley, P.A., Pokric, M., and Thacker, N.A., Calculating Covariances for Mutual Information Coregistration. Proc. MIUA 2004, London, 23-24 September 2004, pp. 77-80.



(a) Translation in x (b) Translation in y (c) Translation in z

(d) Rotation about x (e) Rotation about y (f) Rotation about z

(g) Scaling in x (h) Scaling in y (i) Scaling in z

Figure 1: The standard deviations of the registration parameters. The lines show least-squares linear fits to the data.

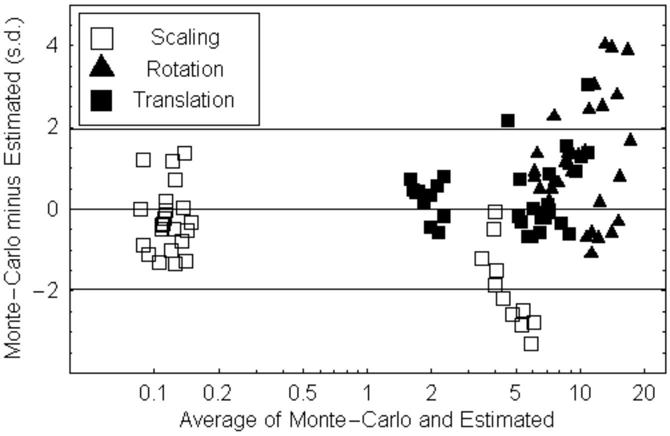


Figure 2: Bland-Altman plots of the estimated and measured (Monte-Carlo) errors on the transformation model parameters. The upper and lower 95% confidence bounds are shown. The units on the abscicca are voxels $\times 10^{-3}$ for the translation and scaling parameters, and degrees $\times 10^{-3}$ for the rotation parameters.

Parameter	Translation	Rotation	Scaling	All
Prob. of agreement	0.755	0.180	0.412	0.781

Table 1: Probabilities of agreement of the estimated and measured (Monte-Carlo) errors on the transformation model parameters, calculated via. a paired t-test. All probabilities are within the 95% confidence limit, indicating that no significant differences between the two error estimates were observed.